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We study the space-time symmetries and transformation properties of the non-commutative U(1) gauge theory, by using Noether charges. We carry out our analysis by keeping an open view on the possible ways  $\theta^{\mu\nu}$  could transform. We conclude that  $\theta^{\mu\nu}$  cannot transform under any space-time transformation since the theory is not invariant under the conformal transformations, with the only exception of space-time translations. The same analysis applies to other gauge groups.

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## I. INTRODUCTION

The idea of non-commuting space-time coordinates was first considered by Heisenberg and written about by Snyder [1]. The geometry was widely developed in recent years by Connes [2]. Following the discovery of Seiberg and Witten [3] of a map (SW map) that relates non-commutative to commutative gauge theories, there has been an increasing interest in studying non-commutativity from a theoretical point of view, as well as its impact on phenomenology [4], [5]. Our purpose is to study the space-time symmetry properties of the U(1) non-commutative gauge theory. The extension of our analysis to other gauge groups is straightforward.

The most common explicit realizations of the non-commutative nature of space-time coordinates are [6]: the canonical structure, the Lie algebra structure, and the  $q$ -deformed space structure. In all cases the algebraic structure is associative.

We shall work with the canonical structure, given by

$$x^\mu * x^\nu - x^\nu * x^\mu = i\theta^{\mu\nu}, \quad (\text{I.1})$$

where the Moyal-Weyl  $*$ -product of two fields  $\phi(x)$  and  $\chi(x)$  is defined as

$$\begin{aligned} (\phi * \chi)(x) &\equiv \exp\left\{\frac{i}{2}\theta^{\mu\nu}\partial_\mu^x\partial_\nu^y\right\}\phi(x)\chi(y)|_{y\rightarrow x} \\ &= \phi(x)\chi(x) + \frac{i}{2}\theta^{\mu\nu}\partial_\mu\phi(x)\partial_\nu\chi(x) - \frac{1}{8}\theta^{\mu\nu}\theta^{\lambda\kappa}\partial_\mu\partial_\lambda\phi(x)\partial_\nu\partial_\kappa\chi(x) + \dots, \end{aligned} \quad (\text{I.2})$$

$\theta^{\mu\nu}$  is  $c$ -number valued, the Greek indices run from 0 to  $n-1$ , and  $n$  is the dimension of the space-time.

The action for the non-commutative Abelian gauge theory in four dimensions is

$$\hat{I} = -\frac{1}{4} \int d^4x \hat{F}^{\mu\nu} \hat{F}_{\mu\nu}, \quad (\text{I.3})$$

where  $\hat{F}_{\mu\nu} = \partial_\mu \hat{A}_\nu - \partial_\nu \hat{A}_\mu - i[\hat{A}_\mu, \hat{A}_\nu]_*$ ,  $\hat{A}_\mu$  can be expressed in terms of a U(1) gauge field  $A_\mu$  and of  $\theta^{\mu\nu}$  by means of the SW map,  $\hat{A}_\mu(A, \theta)$ . Note that  $\hat{A}_\mu(A, \theta) \rightarrow A_\mu$  as  $\theta^{\mu\nu} \rightarrow 0$ , hence, in that limit,  $\hat{F}_{\mu\nu} \rightarrow F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ .

The action (I.3) arises either as the non-commutative analog of a standard gauge theory or as the low energy limit of some string theory [3]. In both cases the physical nature of the gauge field  $A_\mu$  and of  $\theta^{\mu\nu}$  is not clear, and has to be investigated. For instance, one sees that within the  $C^*$  algebra defined by (I.1), the  $*$ -multiplication by  $x_\mu$  (thought as an adjoint action) is a derivation  $\partial_\mu$ . It is this derivative that one makes covariant by introducing the field  $\hat{A}_\mu$  [6]:

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$$x^\mu \rightarrow x^\mu + \theta^{\mu\nu} \hat{A}_\nu \quad (\text{I.4})$$

while, in the standard case, the gauge prescription acts on the momentum  $p_\mu$ :  $p_\mu \rightarrow p_\mu + A_\mu$ . Thus, even if the formal resemblance entitles us to treat  $\hat{A}_\mu$  as the four-potential of electrodynamics [4], [5], one has to perform an open minded investigation of the properties of the theory (I.3). An issue is the nature of  $\theta^{\mu\nu}$ . Some recent investigations have considered  $\theta^{\mu\nu}$  as a second rank tensor under Lorentz transformations, with scaling properties imposed by demanding the action to be scale invariant [7]. In this paper we would like to address the problem of the space-time symmetries and transformation properties of the theory (I.3) from a dynamical perspective, namely by using the Noether charges. This will allow us to have stronger constraints on the possible ways  $\theta^{\mu\nu}$  can transform.

## II. DYNAMICALLY CONSISTENT TRANSFORMATIONS

There are various ways to represent the conformal algebra as transformations acting on fields. An example are the the so-called geometric transformations, namely the effect of the infinitesimal variation of the coordinates

$$x'^\mu - x^\mu = \delta_f x^\mu \equiv -f^\mu(x) , \quad (\text{II.5})$$

on the fields, evaluated at the same point  $x$  (for a review see e.g. [8]). Since we shall be dealing with the action (I.3) we are interested in the transformations of the gauge field and the field strength

$$\delta_f A_\mu \equiv A'_\mu(x) - A_\mu(x) \quad \text{and} \quad \delta_f F_{\mu\nu} \equiv F'_{\mu\nu}(x) - F_{\mu\nu}(x) , \quad (\text{II.6})$$

A separate issue are the transformations of  $\theta^{\mu\nu}$ , on which we shall concentrate in the next Section.

The infinitesimal quantities  $f^\mu$  take the following form

$$f^\mu = a^\mu \quad \text{or} \quad f^\mu = \omega_\nu^\mu x^\nu \quad \text{or} \quad f^\mu = a x^\mu \quad \text{or} \quad f^\mu = a^\mu x^2 - 2a \cdot x x^\mu , \quad (\text{II.7})$$

for infinitesimal translations, rotations (and boosts), dilations, and special conformal transformations, respectively, where, as usual,  $\omega^{\mu\nu} = -\omega^{\nu\mu}$ .

It can be easily shown that the geometric transformations (II.6) can be written as

$$\delta_f A_\mu = \mathbf{L}_f A_\mu \quad \text{and} \quad \delta_f F_{\mu\nu} = \mathbf{L}_f F_{\mu\nu} = \partial_\mu \mathbf{L}_f A_\nu - \partial_\nu \mathbf{L}_f A_\mu , \quad (\text{II.8})$$

where  $\mathbf{L}_f$  is the standard *Lie derivative*

$$\mathbf{L}_f X_{\mu\dots\nu}^{\lambda\dots\kappa} = f^\alpha \partial_\alpha X_{\mu\dots\nu}^{\lambda\dots\kappa} + (\partial_\mu f^\alpha) X_{\alpha\dots\nu}^{\lambda\dots\kappa} + \dots + (\partial_\nu f^\alpha) X_{\mu\dots\alpha}^{\lambda\dots\kappa} - (\partial_\alpha f^\lambda) X_{\mu\dots\nu}^{\alpha\dots\kappa} - \dots - (\partial_\alpha f^\kappa) X_{\mu\dots\nu}^{\lambda\dots\alpha} . \quad (\text{II.9})$$

Note also that one can write the identity

$$\delta_f A_\mu = \mathbf{L}_f A_\mu \equiv f^\alpha F_{\alpha\mu} + D_\mu (f^\alpha A_\alpha) , \quad (\text{II.10})$$

where  $D_\mu = \partial_\mu - iA_\mu$  is the covariant derivative. Since

$$[\delta_f, \delta_g] A_\mu = \delta_{[f,g]} A_\mu \quad \text{and} \quad [\delta_f, \delta_g] F_{\mu\nu} = \delta_{[f,g]} F_{\mu\nu} , \quad (\text{II.11})$$

one sees that indeed this is a representation of the conformal algebra.

On the other hand, one can also introduce the *covariant* geometric transformations [9], namely the transformations  $\bar{\delta}_f$  obtained by gauge-transforming  $\delta_f$ . For instance the transformation of the gauge field given in (II.10) becomes

$$\bar{\delta}_f A_\mu = f^\alpha F_{\alpha\mu} . \quad (\text{II.12})$$

This time one has

$$[\bar{\delta}_f, \bar{\delta}_g] A_\mu = \bar{\delta}_{[f,g]} A_\mu + D_\mu X \quad \text{and} \quad [\bar{\delta}_f, \bar{\delta}_g] F_{\mu\nu} = \bar{\delta}_{[f,g]} F_{\mu\nu} + D_\mu X_\nu , \quad (\text{II.13})$$

thus, as a representation of the original algebra, it closes only up to a gauge transformation, that one is free to perform. For a generalization of the latter to the non-commutative counterparts see [10].

In these two examples there is no need to refer to an action to obtain the way the fields have to transform. In this sense there is a certain amount of arbitrariness, and there is no need for these transformations to be symmetries.

For instance, one can easily check that the scaling properties of fields of arbitrary spin obtained via geometric transformations (either  $\delta_f$  or  $\bar{\delta}_f$ ) do not always agree with the standard scaling properties. Note that this is not the case for  $A_\mu$  and  $F_{\mu\nu}$  in four dimensions, where the geometric scaling properties agree with the standard ones.

Another possible way to represent the conformal algebra acting on the fields is via Noether charges. This (infinite dimensional, reducible) representation has a very important role among the various representations because is the form of the action that dictates the transformations of the fields, whether or not they are symmetries. We shall call these transformations  $\Delta_f$ .

In a theory with Lagrangian density  $\mathcal{L}(\Phi_i, \partial\Phi_i)$  (where the collective index  $i$  takes care of the different fields, as well as their spin type) the Noether current for space-time transformations has the form

$$J_f^\mu = \Pi^{\mu i} \delta_f \Phi_i + \mathcal{L} \delta_f x^\mu, \quad (\text{II.14})$$

where  $\Pi^{\mu i} = \delta\mathcal{L}/\delta\partial_\mu\Phi_i$ , and, as in (II.6),  $\delta_f\Phi_i = \Phi'_i(x) - \Phi_i(x)$ . With the current (II.14) one can: i) test whether the given transformation is a symmetry by picking the correspondent  $f^\mu$  in (II.7), and checking whether  $\partial_\mu J^\mu = 0$ , by using the equations of motion; ii) use the Noether charges  $Q_f \equiv \int d^3x J_f^0$ , and the canonical equal-time Poisson brackets  $\{\Phi_i(x), \Pi^j(y)\} = \delta_i^j \delta(\vec{x} - \vec{y})$ , to generate the transformations of an arbitrary function of the canonical variables

$$\{G(\Phi_i, \Pi_i), Q_f\} \equiv \Delta_f G(\Phi_i, \Pi_i). \quad (\text{II.15})$$

Note also that, for  $f^0 = g^0 = 0$ ,

$$\{Q_f, Q_g\} = Q_{[f,g]}, \quad (\text{II.16})$$

and Eq.s (II.15) and (II.16) hold whether or not  $\partial_0 Q_f = 0$ .

Of course, when  $Q_f$  acts on the fields it must reproduce the transformations one started with  $\Delta_f\Phi_i = \delta_f\Phi_i$ . We call  $\Delta_f$  the *dynamically consistent transformations*.

The expression for the current (II.14) has been obtained by varying the action, including the measure, under an arbitrary space-time transformation, and only afterwards one tests the invariance, following the above described procedure. Sometimes the Noether theorem is used in a somehow different perspective, namely by checking that  $\delta\mathcal{L} = \partial_\mu V^\mu$ , which is only true for invariant actions, and then writing the current as  $J^\mu = \Pi^{\mu i} \delta^* \Phi_i - V^\mu$ , where in this case  $\delta^* \Phi_i = \Phi'_i(x') - \Phi_i(x)$ , hence  $\delta^* \neq \delta_f$ . The choice  $\delta^*$  is the most useful in the case of gauge transformations and supersymmetry [11], while for standard space-time transformations, as the ones we are considering in this paper, it is more convenient to use (II.14). Of course, the two procedures are consistent [11], [12].

### III. NON-COMMUTATIVE U(1) GAUGE THEORY

For sake of clarity, let us now write down the terms in the action in (I.3) up to first order in  $\theta$

$$\hat{I} = -\frac{1}{4} \int d^4x [F^{\mu\nu} F_{\mu\nu} - \frac{1}{2} \theta^{\alpha\beta} F_{\alpha\beta} F^{\mu\nu} F_{\mu\nu} + 2\theta^{\alpha\beta} F_{\alpha\mu} F_{\beta\nu} F^{\mu\nu}] + O(\theta^2), \quad (\text{III.17})$$

although the following results hold to any order in  $\theta$ . The  $\theta$ -dependence in (III.17) is obtained from the  $*$ -product in  $\hat{F}_{\mu\nu}$ , and by using the SW map

$$\hat{A}_\mu(A, \theta) = A_\mu - \frac{1}{2} \theta^{\alpha\beta} A_\alpha (\partial_\beta A_\mu + F_{\beta\mu}) + O(\theta^2), \quad (\text{III.18})$$

which solves the Seiberg-Witten equation [3]

$$\hat{A}(A + \delta_\lambda A) = \hat{A}(A) + \hat{\delta}_\lambda \hat{A}(A), \quad (\text{III.19})$$

where  $\delta_\lambda A_\mu = \partial_\mu \lambda$ ,  $\hat{\delta}_\lambda \hat{A}_\mu = \partial_\mu \hat{\lambda} - i[\hat{A}_\mu, \hat{\lambda}]_*$ , and  $\lambda, \hat{\lambda}(\lambda, A)$  are the parameters of the gauge transformations for standard and non-commutative U(1) gauge group, respectively.

The Noether current for space-time transformations that we obtain from the action (III.17) is then

$$J_f^\mu = \Pi^{\mu\nu} \delta_f A_\nu - \mathcal{L} f^\mu, \quad (\text{III.20})$$

where the  $f^\mu$ 's are given in (II.7),  $\Pi^{\mu\nu} = \delta\mathcal{L}/\delta\partial_\mu A_\nu$ , and, being  $\Pi_{\alpha\beta}^\mu = \delta\mathcal{L}/\delta\partial_\mu\theta^{\alpha\beta} = 0$ , the transformations  $\delta_f\theta^{\alpha\beta}$  do not enter the Noether current.

Let us now analyze the symmetry properties by writing the divergence of this current as

$$\partial_\mu J_f^\mu = \Pi^{\mu\nu} F_{\alpha\nu} \partial_\mu f^\alpha - \mathcal{L} \partial_\mu f^\mu . \quad (\text{III.21})$$

We obtain that  $\partial_\mu J_f^\mu = 0$  for infinitesimal translations  $f^\mu = a^\mu$ , but

$$\partial_\mu J_f^\mu = \omega_\mu^\alpha \Pi^{\mu\nu} F_{\alpha\nu} , \quad \partial_\mu J_f^\mu = a(\Pi^{\mu\nu} F_{\mu\nu} - 4\mathcal{L}) , \quad \partial_\mu J_f^\mu = 2\Pi^{\mu\nu} F_{\alpha\nu} (a^\alpha x_\mu - a_\mu x^\alpha - a \cdot x \delta_\mu^\alpha) + 8a \cdot x \mathcal{L} , \quad (\text{III.22})$$

for infinitesimal Lorentz transformations, dilations and special conformal transformations, respectively. In absence of the non-commutative corrections,  $\theta^{\mu\nu} = 0$ ,  $\Pi^{\mu\nu} = -F^{\mu\nu}$ , and from (III.22) one immediately sees that  $\partial_\mu J_f^\mu = 0$  for all  $f^\mu$ 's in (II.7), and the theory is invariant under the full conformal transformations<sup>1</sup>.

On the other hand, the action  $\hat{I}$ , for  $\theta^{\mu\nu} \neq 0$ , is only invariant under translations. Of course, this leaves room to special choices of the parameters and/or of the field configurations, to obtain conserved currents. For instance, if one performs two *dependent* infinitesimal boosts with parameters  $\omega_1^0 = \omega_2^0 = \omega$ , from the first condition in (III.22) one obtains  $\Pi^{12}(E_2 - E_1) = E_3(\Pi^{13} + \Pi^{23})$ . Thus, for instance, one finds conservation if the electric field  $\vec{E}$  lives in the (1, 2)-plane, and has equal components.

It is crucial to notice that the results in (III.22) hold for the full theory (I.3), to all orders in  $\theta$ . As a matter of fact, to obtain (III.22) we have only used the conditions

$$\partial_\mu \theta = 0 , \quad \Pi^{\mu\nu} = -\Pi^{\nu\mu} , \quad \partial_\mu \Pi^{\mu\nu} = 0 . \quad (\text{III.23})$$

i.e. i)  $\theta^{\mu\nu}$  does not depend on the coordinates  $x^\mu$ ; ii)  $\Pi^{\mu\nu}$  is antisymmetric, which is a consequence of the antisymmetry of  $F_{\mu\nu}$ ; iii) the equations of motion for  $A_\mu$  are satisfied in the form  $\partial_\mu \Pi^{\mu\nu} = 0$ , where the latter expression is valid to all orders in  $\theta$  provided one changes  $\Pi^{\mu\nu}$  accordingly. Furthermore  $\Pi_{\alpha\beta}^\mu = 0$  to all orders.

We now want to check the dynamical consistency of the transformations, along the lines of what explained in Section II. It is straightforward to see that for  $A_\mu$  and  $F_{\mu\nu}$

$$\Delta_f A_\mu = \delta_f A_\mu \quad \text{and} \quad \Delta_f F_{\mu\nu} = \delta_f F_{\mu\nu} , \quad (\text{III.24})$$

while for  $\theta^{\mu\nu}$

$$\Delta_f \theta^{\mu\nu} = 0 , \quad (\text{III.25})$$

for all  $f^\mu$ 's in (II.7).

At this point one has to investigate whether  $\theta^{\mu\nu}$  could be treated as a dummy field  $D$ , in the spirit of what happens in supersymmetric models. The answer is no. In both cases  $\delta_f \theta^{\mu\nu}$  and  $\delta_{\text{SUSY}} D$  do not explicitly enter the expression of the Noether current, hence one can arbitrarily choose them. Nonetheless, in supersymmetric models: i) one fixes  $\delta_{\text{SUSY}} D$  by demanding the action  $I_{\text{SUSY}}$  (which contains the dummy fields explicitly) to be invariant; ii) even if  $\Pi_D = 0$ ,  $\Delta_{\text{SUSY}} D \neq 0$  due to the fact that  $D$  can be properly expressed in terms of the dynamical fields by means of the constraint  $\delta I_{\text{SUSY}}/\delta D = 0$  [11]. Therefore  $\delta_{\text{SUSY}}$  represents the SUSY algebra *off-shell*, while  $\Delta_{\text{SUSY}}$  represents the SUSY algebra *on-shell*, and both are symmetries of the action. On the contrary, in our case, if one uses

$$\frac{\delta \hat{I}}{\delta \theta^{\mu\nu}} = 0 , \quad (\text{III.26})$$

as a constraint, then one obtains different dynamical transformations of  $\theta^{\mu\nu}$  at different orders, and trivial theories. For instance, for the first order action (III.17), one cannot express  $\theta^{\mu\nu}$  in terms of the dynamical fields, hence there is no contribution to  $\Delta_f \theta^{\mu\nu}$  from  $\Delta_f A_\mu$  or  $\Delta_f F_{\mu\nu}$ . While, at higher orders, there are not simple expressions for  $\theta^{\mu\nu}$  in terms of  $A_\mu$  or  $F_{\mu\nu}$ , and  $\Delta_f \theta^{\mu\nu}$  has different expressions for different orders. Furthermore, the constraints imposed on  $F_{\mu\nu}$  by (III.26) make the theory trivial. For instance, at first order, (III.26) implies  $F_{\mu\nu} = 0$ , which is too trivial

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<sup>1</sup>Since  $A_\mu$  is a spin 1 field, and we are in four dimensions, for  $\theta^{\mu\nu} = 0$ , scale invariance alone implies special conformal invariance [13].

a theory. Note also that if one uses (III.26) the expressions of  $\partial_\mu J_f^\mu$  in (III.22) are trivially zero for all conformal transformations.

We conclude that  $\delta_f \theta^{\mu\nu}$  cannot be fixed by any symmetry requirement (with the only exception of translations), and that to have a physical meaningful theory one should not make use of the "equations of motion" (III.26). Therefore, among all possible  $\delta_f \theta^{\mu\nu}$ 's that represent the conformal algebra, the most natural choice seems to us  $\delta_f \theta^{\mu\nu} = \Delta_f \theta^{\mu\nu} = 0$  (which agrees with the translation invariance), and  $\theta^{\mu\nu}$  does not transform under dynamically consistent space-time transformations. This choice is also consistent with considerations in [10].

#### IV. CONCLUSIONS

We have studied the space-time symmetries and transformation properties of the non-commutative U(1) gauge theory, from the dynamical point of view. The same analysis applies to other gauge groups.

We carried out our analysis by keeping an open view on the possible ways  $\theta^{\mu\nu}$  could transform. A possible scenario is the one with  $\theta^{\mu\nu}$  behaving like a second rank tensor under Lorentz transformations [7]. Another possibility is to see  $\theta^{\mu\nu}$  as a back-ground metric, therefore reduce the space-time transformations to the isometries of this metric which would leave the theory invariant.

From our analysis it clearly follows that there is no reason for  $\theta^{\mu\nu}$  to transform under any space-time transformation since the theory is not invariant under the conformal transformations, with the only exception of translations, and special choices of the parameters and/or of the field configurations. This implies that  $\theta^{\mu\nu}$  is an invariant constant matrix in any frame of reference.

Since there is no invariant constant antisymmetric second rank tensor in four dimensions, the physics necessarily depends on the frame of reference. Examples, for special choices of  $\theta^{\mu\nu}$ , are given in [4] and [5], where it is shown that, in such a theory, the velocity of light depends on the direction of motion, like in an anisotropic medium.

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